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OPERATION OF DISTRIBUTED GENERATION UNDER STOCHASTIC PRICES

AFZAL SIDDIQUI AND CHRIS MARNAY

Abstract. We model the operating decisions of a commercial enterprise that needs to satisfy its periodic electricity demand with either on-site distributed generation (DG) or purchases from the wholesale market. While the former option involves electricity generation at relatively high and possibly stochastic costs from a set of capacity-constrained DG technologies, the latter implies unlimited open-market transactions at stochastic prices. A stochastic dynamic programme (SDP) is used to solve the resulting optimisation problem. By solving the SDP with and without the availability of DG units, the implied option values of the DG units are obtained.

1. Background

The deregulation of electricity industries worldwide has brought market forces into a sector that was once state regulated. This liberalisation has often resulted in the divestiture of utility-owned power plants to private companies that will use these assets to sell power at volatile market prices. Consequently, market-based methods are necessary in the deregulated environment to value power plants and price the various electricity products. Towards this end, techniques developed to price financial instruments, such as options, have been applied to electricity markets. For example, [4] finds a closed-form solution for the value of a power plant under the assumption that operating constraints are not significant and both electricity and gas prices evolve randomly. The analysis is done for both correlated geometric Brownian motion (GBM) and Ornstein-Uhlenbeck (OU) specifications.[1] This work was extended in [5] to incorporate operating constraints and finds that the power plant is over-valued more under an assumption of OU prices because the operational constraints together with the lower spark spread, resulting from mean-reverting prices, force the plant to be turned on and off more frequent. It, therefore, incurs greater start-up costs than under an assumption of GBM prices. Furthermore, more efficient plants have higher values under OU rather than GBM assumptions (and *vice versa*) since the lower heat rates associated with higher efficiency more than compensate for the more frequent changes in operating levels necessitated by mean-reverting prices. On the other hand, the higher heat rates for less efficient plants valued under OU assumptions exacerbate the consequences of the required changes in operating levels. In this case, a lattice-based approach was taken that discretises the underlying stochastic processes and writes a stochastic

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[1]In the former process, successive percentage changes in the value of one quantity are independent of each other, while in the latter, the values tend to revert to long-term mean. While the OU process is more representative of energy prices, the GBM one is more straightforward to model.

dynamic programme (SDP) on them that is solved using backward induction (see [3] or [9]) for the development of one-dimensional lattices under the assumption of GBM, [1] or [2] for multinomial lattice techniques involving GBMs, and [6] for a lattice technique to model a combined GBM-OU process). Finally, [8] and [12] use this real options framework for investment and short-term operation implications, respectively.

While deregulation has liberalised the rules governing plant ownership, it has also opened up the possibility for distributed generation (DG) to be installed. Such small-scale, on-site generators could potentially provide a more efficient means for end-use consumers to meet their energy loads than traditional central generation (see [10] and [11] for customer adoption and operation of DG in deterministic settings). With this alternative available to the traditional paradigm, how would a rational investor value a portfolio of generators rather than individual power plants? This paper addresses the following problem: imagine that you must meet a deterministic load during each period of a month via either electricity purchases at stochastic spot market prices or a portfolio of DG units. The spot market price evolves according to a GBM, whereas the costs of the DG units are relatively high and can be either deterministic or stochastic according to a GBM correlated with the spot market price as well. In this instance, we use a portfolio of two DG units, one with stochastic generating costs and the other with deterministic ones. Effectively, the former can be thought of as a reciprocating engine running on natural gas, whereas the latter could be a more costly technology such as a fuel cell (FC). This is a simple model in which a linear programme is solved at each node of the resulting lattice to minimise the expected cost of meeting the annual load.

The objective of this study, therefore, is to determine how the implied option values of the DG units change if the NG price is allowed to be stochastic. Furthermore, it addresses whether a portfolio of options on the underlying DG units can be used to approximate the option value of a portfolio of DG units. The effects of operational constraints and stochastic NG prices on this discrepancy will also be analysed.

2. Mathematical Formulation

2.1. **Parameters.** We begin by defining the following parameters:

- T: time horizon in years
- Δt: length of time interval in years
- $N = \frac{T}{\Delta t}$: number of decision-making steps over the time horizon
- $s(k, i)$: spot market price in €/kWh during period k given that there have been i upward movements (in the spot market price), where $0 \leq k \leq N$ and $0 \leq i \leq k$
- $c_{NG}(k, j)$: cost in €/kWh of using the reciprocating engine during period k given that there have been j upward movements (in the NG price), where $0 \leq k \leq N$ and $0 \leq i \leq k$
- c_{FC}: deterministic cost in €/kWh of using the FC
- p_1: risk-neutral probability of upward movements in both the spot market price and the generating cost of the reciprocating engine
- p_2: risk-neutral probability of an upward movement in the spot market price and a downward movement in the generating cost of the reciprocating engine

- p_3: risk-neutral probability of a downward movement in the spot market price and an upward movement in the generating cost of the reciprocating engine
- p_4: risk-neutral probability of downward movements in both the spot market price and the generating cost of the reciprocating engine
- σ_1: standard deviation of the percentage changes in the spot market price process
- σ_2: standard deviation of the percentage changes in the reciprocating engine generating cost process
- $u_1 = e^{\sigma_1\sqrt{\Delta t}}$: jump size for the spot market price process
- $u_2 = e^{\sigma_2\sqrt{\Delta t}}$: jump size for the reciprocating engine generating cost process
- ρ: the degree of correlation between the spot market price and the reciprocating engine generating cost
- $\ell(k)$: electricity demand in kWh during period k
- $\overline{z}(m)$: maximum capacity in kW of DG unit m, where $m \in \{NG, FC\}$
- $0 \leq \underline{z}(m) \leq \overline{z}(m)$: minimum capacity in kW of DG unit m, where $m \in \{NG, FC\}$
- α: risk-free interest rate per annum
- $\mu_1 = \alpha - 0.5\sigma_1^2$: risk-free drift rate for the spot market price process
- $\mu_2 = \alpha - 0.5\sigma_2^2$: risk-free drift rate for the reciprocating engine generating cost process
- $\beta = e^{-\alpha\Delta t}$: discount factor over one time step
- $\upsilon = 8760$: number of hours per year

2.2. **Price Processes.** We now assume that both the electricity spot market price and the NG price follow correlated GBM processes as follows:

$$\begin{aligned} ds_t &= \mu_1 s_t dt + \sigma_1 s_t dz_s \\ dc_t &= \mu_2 c_t dt + \sigma_2 c_t dz_c \end{aligned}$$

Here, dz_s and dz_c are GBMs with instananeous correlation ρ. Using the approach of [2], these continuous-time processes can be approximated by discrete jumps so that given the price vector at time k for i upward movements in the spot market price and j upward movements in the NG price, $(s(k,i), c_{NG}(k,j))$, there are four possible states to reach by time $k+1$:

$$\begin{aligned} \{&(s(k,i)u_1, c_{NG}(k,j)u_2), \\ &(s(k,i)u_1, c_{NG}(k,j)d_2), \\ &(s(k,i)d_1, c_{NG}(k,j)u_2), \\ &(s(k,i)d_1, c_{NG}(k,j)d_2)\} \end{aligned}$$

In effect, the two correlated price processes are discretised along a multinomial lattice in which the prices at any state (i,j) are $s(k,i) = s(0,0)u_1^i d_1^{k-i}$ and $c_{NG}(k,j) = c_{NG}(0,0)u_2^j d_2^{k-j}$, where $d_1 \equiv 1/u_1$ and $d_2 \equiv 1/u_2$. The risk-neutral probabilities of movements in the lattice are indicated as follows:

$$p_1 = \frac{1}{4}\left(1 + \rho + \sqrt{\Delta t}\left(\frac{\mu_1}{\sigma_1} + \frac{\mu_2}{\sigma_2}\right)\right)$$
$$p_2 = \frac{1}{4}\left(1 + \rho + \sqrt{\Delta t}\left(\frac{\mu_1}{\sigma_1} - \frac{\mu_2}{\sigma_2}\right)\right)$$
$$p_3 = \frac{1}{4}\left(1 + \rho + \sqrt{\Delta t}\left(\frac{-\mu_1}{\sigma_1} + \frac{\mu_2}{\sigma_2}\right)\right)$$
$$p_4 = \frac{1}{4}\left(1 + \rho + \sqrt{\Delta t}\left(\frac{-\mu_1}{\sigma_1} - \frac{\mu_2}{\sigma_2}\right)\right)$$

2.3. **Decision Variables.** The decision variables needed for the SDP are as follows:

- $x(k, i, j)$: electricity capacity purchased in kW from spot market during period k in state (i, j)
- $y(k, m, i, j)$: electricity capacity provided in kW by DG unit m during period k in state (i, j)

2.4. **Value Function.** We define the value function for the SDP as follows:

- $V(k, i, j)$: minimum expected discounted cost to go in period k given that i upward steps in the spot market price and j upward steps in the reciprocating engine cost process have occurred

2.5. **SDP.** A SDP is now formulated to minimise the expected discounted cost of meeting the electricity load given the capacity constraints of the DG units and a prohibition against selling back to the grid:

- SDP recursion for $0 \leq k < N$, $0 \leq i \leq k$, and $0 \leq j \leq k$:

$$\begin{aligned} V(k,i,j) \quad = \quad & \min_{x(k,i,j),y(k,m,i,j)} \{s(k,i)x(k,i,j)\Delta tv + c_{NG}(k,j)y(k,NG,i,j)\Delta tv \\ & + c_{FC}y(k,FC,i,j)\Delta tv | x(k,i,j)\Delta tv + y(k,NG,i,j)\Delta tv \\ & + y(k,FC,i,j)\Delta tv = \ell(k), \underline{z}(m) \leq y(k,m,i,j) \leq \overline{z}(m), x(k,i,j) \geq 0\} \\ & + \beta\{p_1 V(k+1,i+1,j+1) + p_2 V(k+1,i+1,j) \\ & + p_3 V(k+1,i,j+1) + p_4 V(k+1,i,j)\} \end{aligned}$$

- Terminal condition for $0 \leq i \leq N$ and $0 \leq j \leq N$:

$$V(N, i, j) = 0$$

- Answer:

$$V(0, 0, 0)$$

At each node of the multinomial lattice, the SDP selects the minimum-cost way to meet the electricity demand during the corresponding period via spot market purchases or constrained DG units and adds to this cost the expected discounted cost of following such an optimal policy in subsequent periods. It is solved using backward induction by starting at period N.

3. Numerical Example

In order to illustrate the method, we implement it for some hypothetical data. We assume that the time horizon to be analysed is thirty days long, i.e., $T = \frac{30}{365}$, and that a dispatch decision is made on a daily basis, i.e., $N = 30$ or the length of each time step, Δt, is twenty-four hours. The initial spot market price of electricity is $s(0,0) =$ €0.045/kWh and that two DG units are installed: one is a NG-fired reciprocating engine with a capacity of 200kW and initial operating cost of $c_{NG}(0,0) =$ €0.0475/kWh, and the other is a FC with a capacity of 100kW and a constant operating cost of $c_{FC} =$ €0.0525/kWh. The risk-free interest rate is 2.4% per annum, and the volatilities of the electricity and NG prices are both 98% per annum, although we allow that of electricity to increase up to a level of 196% per annum.[2] In addition, we assume a correlation coefficient of 0.50 between the electricity and NG prices. Finally, the electricity demand is indicated in Figure 1.

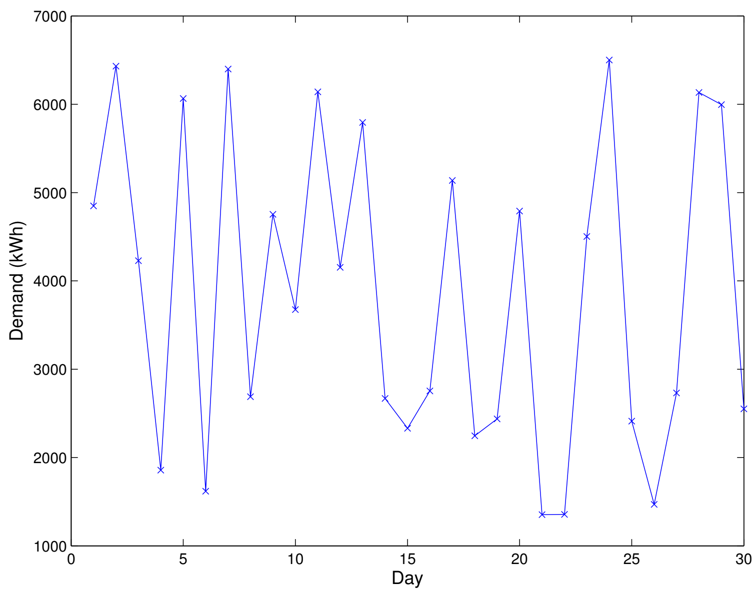

Figure 1. Daily Demand

The implied option value of each DG unit and the portfolio of units is determined by first solving the SDP with all DG units switched off and then subtracting from this cost the value of the objective function when the DG unit(s) is (are) allowed to operate. The option value per kW is then calculated by dividing the cost savings under that scenario by the total DG capacity available. For example, in order to find the implied option value per kW of the NG unit, we first run the model with the NG unit switched off and find the minimised cost. Next, we run the model with the NG unit available up to its capacity limit and find the minimised cost under this scenario. Finally, we divided the cost difference between these two runs by the total available capacity, i.e., 200kW, to determine the implied option value per kW. We

[2]Such high volatilities are necessary over such a short time horizon, otherwise the DG units will never be deployed.

perform this analysis and find that the implied option value of the NG unit increases with electricity price volatility as expected, although the value is decreased if the NG price is allowed to be stochastic (see Figure 2). Intuitively, as the electricity price becomes more volatile, it becomes more likely that it will exceed the NG "strike price." Consequently, the NG unit will be dispatched more frequently. However, if the NG price is stochastic as well, then due to the positive correlation between it and the electricity price, there will be fewer nodes in the lattice at which it will be beneficial to dispatch the NG unit. As a result, the implied option value will be smaller than in the deterministic NG case. Nevertheless, increasing the volatility of the electricity price while keeping that of the NG price constant will still result in a monotonically increasing implied option value.

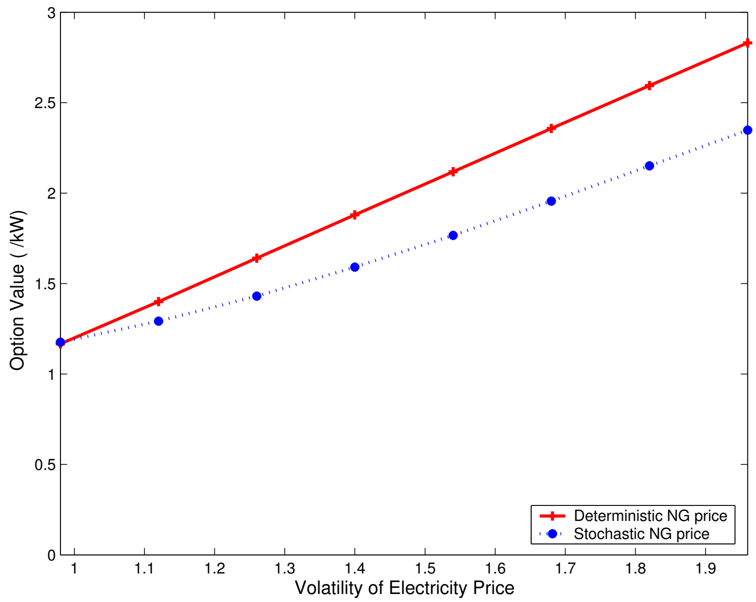

FIGURE 2. Implied Option Value of the NG Unit

If the 200kW NG unit is installed in combination with a 100kW FC, then the implied option value of the portfolio of generators can be determined analogously. It is important to note, however, that this value will generally not be the same as the weighted (by the capacities) average of the implied option values of the individual DG units. In fact, the weighted average will usually overstate the implied option value of the portfolio of DG units because the former technique does not recognise that the cheaper DG unit, i.e., usually the NG one, will "crowd out" the more expensive one. Therefore, some of the value of the 100kW FC will be lost as the 200kW NG unit will always be dispatched ahead of it in case of high electricity prices. For example, if NG prices are deterministic and the electricity price has a volatility of 98%, then the implied option values for the NG unit and FC are €1.17/kW and €0.76/kW, respectively, resulting in a capacity-weighted average value of €1.03/kW.[3] The implied option value when these two DG units

[3]This is simply $\frac{200*1.17+100*0.76}{200+100}$.

are operated together in a portfolio, however, is €0.82/kW, thereby implying an overstatement of 26% if the capacity-weighted approximation is used. Furthermore, this overstatement increases with the volatility of the electricity price because higher volatility implies that the NG unit will be more likely to be dispatched, and hence, more likely to "crowd out" the FC. Finally, this "crowding out" effect is at a decreasing rate because as the electricity price volatility becomes very high, the capacity limit on the NG unit is reached, which eliminates its ability to affect the dispatch of the FC (see Figure 3).

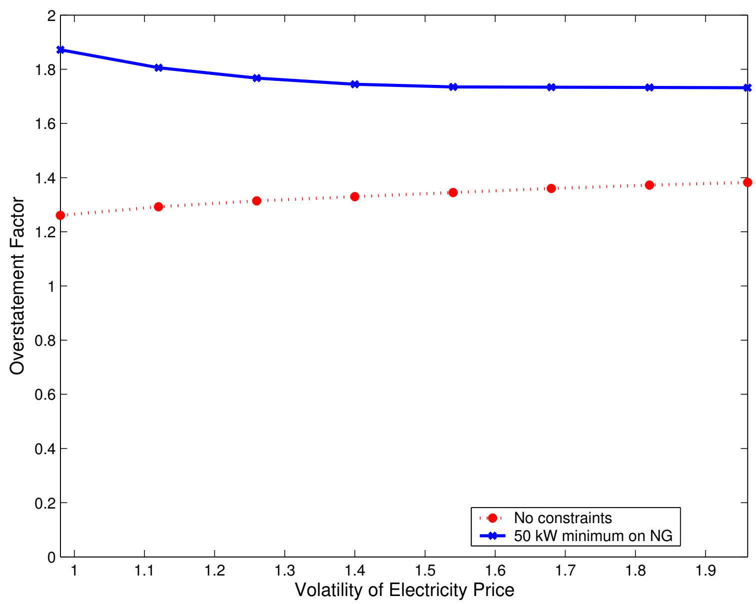

FIGURE 3. Overstatement of the Implied Option Value of a Portfolio of DG Units Under Deterministic NG Prices

This discrepancy between the implied option value of a portfolio of DG units and the capacity-weighted approximation increases if the NG unit is forced to run at a minimum of 50kW. For example, for electricity price volatility of 98%, the implied option values for the NG unit under the minimum run-time constraint and FC are €0.31/kW and €0.76/kW, respectively, resulting in a capacity-weighted average value of €0.46/kW, whereas the implied option value of the portfolio is €0.25/kW. Now, the lower bound on NG generation reduces value by removing some flexibility from operations, i.e., by forcing the NG unit to be running even when it is not economical, which causes the overstatement to be higher than in the unconstrained case. Unlike the unconstrained case, however, the overstatement here decreases with electricity price volatility because the "crowding out" effect does not strengthen. Indeed, if the electricity price volatility increases from 96% to 112%, the NG unit will not be able to reduce the FC's share of generation because the latter was already so low. At the same time, this increase in electricity price volatility increases the implied option value of the portfolio of DG units, thereby decreasing the overstatement (see Figure 3).

Under stochastic NG prices, the overstatement with the capacity-weighted approximation is reduced vis à vis the case with deterministic NG prices. This follows because uncertain NG prices imply that the NG unit will be less likely to be economical relative to market-procured electricity. Therefore, it will be less likely to "crowd out" the FC, thereby reducing the magnitude of the overstatement. Similar to the deterministic case, a 50kW minimum run-time constraint on the NG unit increases the magnitude of the overstatement. In contrast to the deterministic case, here, the uncertainty in the NG price does not result in immediate "crowding out" of the FC unit. Therefore, increases in the electricity price volatility cause it to be more "crowded out," which then reduces the value of the portfolio to DG units relative to the capacity-weighted approximation value (see Figure 4). Hence, using the capacity-weighted approximation to calculate the implied option value of a portfolio of DG units is more accurate when the NG prices are modelled as being stochastic.

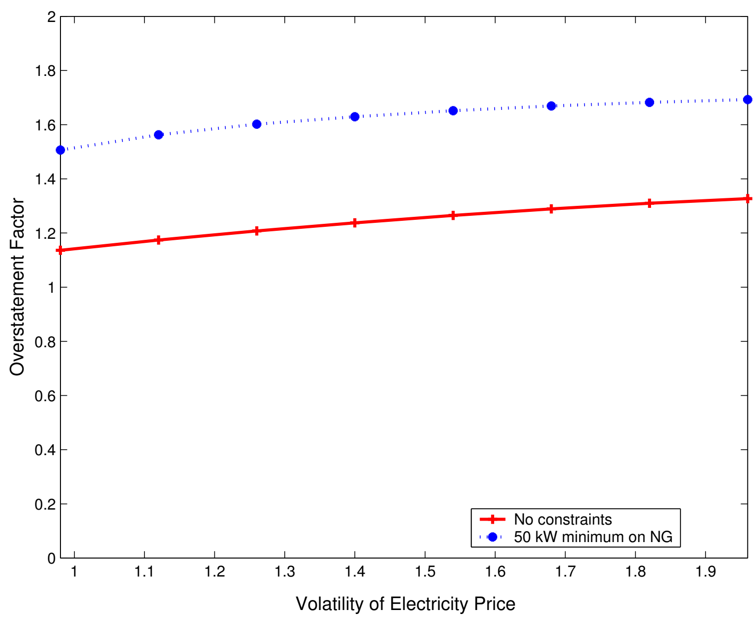

FIGURE 4. Overstatement of the Implied Option Value of a Portfolio of DG Units Under Stochastic NG Prices

4. DISCUSSION

In this paper, we develop a simple model to determine the implied option value of a portfolio of DG units. We find that the value of any individual unit increases with electricity price volatility and is reduced if the fuel price for that unit is stochastic. Furthermore, we find that using a capacity-weighted portfolio of options overstates the implied option value of the portfolio because it ignores the "crowding out" of the more expensive DG unit by the less expensive one. This effect is amplified with the imposition of minimum run-time constraints and mitigated when the input fuel price is stochastic. Intuitively, the former reduces the flexibility of the DG units, and the latter reduces the "crowding out" effect.

For future work, it would be interesting to explore a longer time horizon than one month. Indeed, analysing the investment decision would involve a more long-term study. In this case, the multinomial lattice technique would be computationally infeasible, and least-squares Monte Carlo simulation, as developed in [7], would be required. Other enhancements should include a mean-reverting OU process for the logarithm of the spot market price and other operational constraints, such as ramping times, start-up costs, and variable heat rates.

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